LOW-TEMPERATURE THERMOMAGNETIC PHENOMENA IN METALS UNDER JOULE HEATING CONDITIONS

V. R. Sobol', O. N. Mazurenko, and
V. K. Solonovich

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The problem of the temperature distribution over the cross section of rectangular samples of single crystals of transition metals under Joule heating conditions is considered analytically. The role of a transverse magnetic field in the formation of the temperature field in anisotropic heat exchange with the environment is determined.

It is known that transfer phenomena of both charge and heat under low-temperature conditions are determined, to a considerable degree, by the electronic subsystem, namely, the dispersion laws of conduction electrons, their mechanisms of scattering by defects of the crystalline structure [1–4]. Investigation of transfer phenomena and kinetic coefficients under isothermal or adiabatic conditions requires special means of coupling the sample with a thermostat that ensure anisotropy of heat exchange with the environment. In galvanomagnetic investigations in the presence of finite densities of the excitation current such circumstances may cause uncontrollable disturbance of the thermal regime of the system and, as a consequence, thr asppearance of thermoelectric fields. In [4] the authors determined the extent of the mutual influence of galvano- and thermomagnetic phenomena in normal uncompensated metals with closed and open Fermi surfaces (FS) under conditions of no heat flux along the magnetic field.

The present work is devoted to consideration of a similar stationary thermal problem for metals obeying a different dispersion law. We have in mind compensated metals, with are characterized by a fundamentally different behavior of the kinetic coefficients in a strong magnetic field, which leads to high levels of dissipation [5, 6]. Along with symmetric heat removal it is of interest to study the influence of the strong "Hall" drift of the carriers in a magnetic field on the symmetry of the heat distribution with respect to the cross section.

Thermal problems for thin metallic samples are traditionally based on the approximation of the absence of any temperature distribution over the volume because of the high value of a kinetic coefficient such as the thermal conductivity and the smallness of the cross section [7]. In the present case the notion of smallness of the cross section is complicated by the fact that under conditions of strong anisotropy of the kinetic coefficients, anisotropy of heat removal, and the need to decrease the influence of dimensional effects low-temperature transfer phenomena may exert a mutual influence on each other.

As the object for consideration we have chosen a sample with a square cross section, along the long axis of which an electric current of density j_x flows. The sample was placed in liquid helium. One pair of lateral faces, normal to the vector of the magnetic intensity $\mathbf{H} = H_z$, was open, thus allowing heat exchange with a thermostat to proceed, while another pair of faces, parallel to $\mathbf{H}$, was adiabatically insulated. The problem will be solved under the assumption that elastic electron-impurity scattering serves as the main mechanism of dissipation. This will permit us to use, in analysis, the closed theory of transfer phenomena in a magnetic field and to adequately describe relaxation processes under conditions of helium temperatures.

The temperature distribution over the cross section in the static case satisfies the differential heat transfer equation

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Institute of Solid-State and Semiconductor Physics, Academy of Sciences of Belarus, Minsk, Belarus.
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$$\mathbf{jE} - \operatorname{div} \mathbf{q} = 0\,, \tag{1}$$

supplemented by the generalized equations of charge and heat transfer

$$E_i = \rho_{ik} j_k + \alpha_{ik} \frac{\partial T}{\partial x_k}, \quad q_i = \pi_{ik} j_k - \kappa_{ik} \frac{\partial T}{\partial x_k}. \tag{2}$$

Here E_i and q_i are components of the electric field $\mathbf{E}$ and the heat flux $\mathbf{q}$ in the direction of the external normal; ρ_{ik}, α_{ik}, and κ_{ik} are components of the electric resistivity, thermoelectromotive force, and heat conduction tensors; π_{ik} is a component of the tensor describing the Peltier effect; $i, k = x, y, z$, summation is carried out with respect to recurring indices; $\partial T/\partial x_k$ is the temperature gradient along one of the directions.

Since the sample dimensions along the electric current direction are larger than the transverse dimensions, we may use the approximation of a long sample without a temperature distribution along the OX axis. Then, eliminating the gradient $\partial T/\partial y$ from the expression for the flux component q_z with account for the adopted τ-approximation, within the framework of which the kinetic coefficients depend on the temperature according to a simple linear or square law, we obtain from (1) and (2) an expression for the temperature distribution along the OZ axis:

$$\frac{\partial^2 T^2}{\partial z^2} + a\frac{\partial T^2}{\partial z} + dT^2 + b = 0\,, \tag{3}$$

$$a = \left[\alpha_{xz} - \left(\pi_{zx} - \kappa_{zy}\frac{\pi_{yx}}{\kappa_{yy}}\right)' - \alpha_{xy}\frac{\kappa_{yz}}{\kappa_{yy}}\right]' \left[\left(\kappa_{zz} - \frac{\kappa_{zy}\kappa_{yz}}{\kappa_{yy}}\right)'\right]^{-1} j_x\,, \tag{4}$$

$$d = (\alpha_{xy}\kappa_{yy}^{-1}\pi_{yx})''\,[(\kappa_{zz} - \kappa_{yz}\kappa_{zy}\kappa_{yy}^{-1})']^{-1} j_x^2\,, \tag{5}$$

$$b = \rho_{xx} j_x^2\,[(\kappa_{zz} - \kappa_{yz}\kappa_{zy}\kappa_{yy}^{-1})']^{-1}\,, \tag{6}$$

here a prime denotes differentiation with respect to temperature.

The equation obtained is the second-order equation with constant coefficients, whose solution, using the Lagrange method, is as follows:

$$T = \left[c_1 \exp(\lambda_1 z) + c_2 \exp(\lambda_2 z) - \frac{b}{\lambda_1 \lambda_2}\right]^{0.5}. \tag{7}$$

Here λ_1 and λ_2 are the roots of characteristic equation (3).

The form of the solution is essentially determined by the type of the roots λ_1 and λ_2, which depend on the relationship between the parameters a^2 and $4d$ with respect to both absolute value and sign. Using the symmetry conditions of the kinetic coefficients in a strong magnetic field [8] and expressions for the tensors $\hat{\rho}$, $\hat{\kappa}$, $\hat{\beta}$, $\hat{\pi}$:

$$\alpha_{ik} = -\rho_{il}\beta_{lk}; \quad \pi_{ik} = -T\beta_{il}\rho_{lk}\,, \tag{8}$$

where β_{lk} is the proportionality coefficient in the relation

$$j_i = \sigma_{ik} E_k + \beta_{ik}\frac{\partial T}{\partial x_k}\,,$$

we obtain

$$\alpha_{xz} \simeq -\omega\tau\,|\alpha|\,,\quad \pi_{zx} \simeq -\omega\tau\,|\pi|\,,\quad \kappa_{yz} \simeq (\omega\tau)^{-1}\,|\kappa|\,,\quad \kappa_{zz} \simeq |\kappa|\,,$$
$$\pi_{yx} \simeq -\omega\tau\,|\pi|\,,\quad \kappa_{zy} \simeq -(\omega\tau)^{-1}\,|\kappa|\,,\quad \alpha_{xy} \simeq -\omega\tau\,|\alpha|\,,\quad \kappa_{yy} \simeq (\omega\tau)^{-2}\,|\kappa|\,, \tag{9}$$

here α, κ, π are the kinetic coefficients in the absence of a magnetic field; ω is the cyclotron frequency; τ is the relaxation time.

As follows from (4), the coefficient a is determined by a sum of four summands. Since the last two summands in (4) differ only in the tensor components α'_{xy} and π''_{yx} and the form of these components is such that by virtue of the symmetry conditions they are expressed in terms of the same components of $\hat{\beta}$ and $\hat{\rho}$, it may be argued that these summands are asymptotically equal in absolute value and, as a consequence, $a = -\omega\tau|\alpha''|\,|\kappa'|^{-1}j_x$. It is characteristic that the sign of this coefficient is determined with an accuracy up to the direction of the current and the magnetic field.

The second parameter needed to solve the characteristic equation is the coefficient d, whose magnitude and sign are determined by the product $\alpha'_{xy}\pi''_{yx}$. Using relations (8) and (9), it is easily to verify that

$$d = -(\omega\tau)^4\,|\alpha'|\,|\pi''|\,|\kappa'|^{-2}j_x^2\,;\quad \frac{d}{a^2} \simeq (\omega\tau)^2$$

and at $\omega\tau >> 1$, $\lambda_{1,2} \simeq \pm\, d^{0.5} = \pm\,\lambda$.

In accordance with the boundary conditions, when the temperature of the open faces is T_0 and the gradient along the OZ axis at the sample center vanishes because of the symmetry of the problem: $T(z = l) = T(z = -l) = T_0$; $\partial T/\partial z|_{z=0} = 0$, we obtain

$$T = \left\{ \left(T_0^2 - \frac{b}{d} \right) [\exp(\lambda l) + \exp(-\lambda l)]^{-1}\, [\exp(\lambda z) + \exp(-\lambda z)] + \frac{b}{d} \right\}^{0.5}. \tag{10}$$

Differentiating of the obtained expression with respect to z, we arrive at an expression for the temperature gradient along the OZ axis

$$\frac{\partial T}{\partial z} = \left\{ \left(T_0^2 - \frac{b}{d} \right) [\exp(\lambda l) + \exp(-\lambda l)]^{-1}\, [\exp(\lambda z) + \exp(-\lambda z)] + \frac{b}{d} \right\}^{-0.5} \times$$
$$\times \left\{ \lambda \left(T_0^2 - \frac{b}{d} \right) [\exp(\lambda l) + \exp(-\lambda l)]^{-1}\, [\exp(\lambda z) - \exp(-\lambda z)] \right\}. \tag{11}$$

We simplify this expression by expanding the exponents and retaining the terms with the maximum values

$$\frac{\partial T}{\partial z} = (T_0 d - T_0^{-1} b)\, z\,, \tag{12}$$

Here, the coordinate dependence of the temperature is a square-law dependence. In accordance with (5) the first term in expression (12) is negative in sign and even with respect to the current direction. To find a quantitative form of gradient (12), it is necessary to evaluate the coefficients d and b.

It is characteristic that the gradient $\partial T/\partial z$ is an even function of the magnetic field and the current. This is clear from the viewpoint of general physics, since a change in the sign of the temperature gradient would contradict general principles of the thermodynamics of inversible processes.

The adiabatic condition along the OY axis does not exclude a temperature distribution along this direction. In this case the temperature gradient is as follows:

$$\frac{\partial T}{\partial y} = \kappa_{yy}^{-1} \left(\pi_{yx} j_x - \kappa_{yz} \frac{\partial T}{\partial z} \right). \tag{13}$$

The dependence of the temperature on the variable y is linear in the present statement of the problem. At the sample center the gradient is determined by the Ettingshausen effect, while throughout the volume its form depends on the scale of the parameters entering expression (13).

Since the Nernst thermoelectromotive force is

$$(E_N)_x = \alpha_{xy} \frac{\partial T}{\partial y} \simeq \frac{(\omega\tau)^4}{\kappa'} (\alpha')^2 T j_x ,$$

then the thermoelectric component is an even function of the direction of the magnetic field vector and an odd function of the transport current, which makes it difficult to separate this component from the ohmic component upon inversion of the excitation current.

For completeness of the analysis we shall consider the situation where the pair of faces normal to the vector of the magnetic field intensity are adiabatically insulated, the heat flux along the OZ axis vanishes, and heat transfer occurs through the faces parallel to the magnetic field vector. Eliminating the gradient $\partial T/\partial Z$ from expression (2) for the heat flux component q_g and transforming Eq. (1), it is easily to verify that if the direction of the transport current remains unchanged then in order to write the solution for the temperature distribution along the OY axis, it is necessary to interchange the subscripts y, z in expressions (3)-(8) without changing the subscript x of the components of the tensors $\hat{\alpha}$, $\hat{\rho}$, $\hat{\pi}$. Then the temperature distribution along the OY axis will satisfy an equation analogous to (3):

$$\frac{\partial^2 T^2}{\partial y^2} + a \frac{\partial T^2}{\partial y} + dT^2 + b = 0 , \tag{14}$$

$$a = \left[\alpha_{xy} - \left(\pi_{yx} - \kappa_{yz} \frac{\pi_{zx}}{\kappa_{zz}} \right)' - \alpha_{xz} \frac{\kappa_{zy}}{\kappa_{zz}} \right]' j_x \left[\left(\kappa_{yy} - \frac{\kappa_{zy}\kappa_{yz}}{\kappa_{zz}} \right)' \right]^{-1} , \tag{15}$$

$$d = (\alpha_{xz} \kappa_{zz}^{-1} \pi_{zx})'' \, [(\kappa_{yy} - \kappa_{zy} \kappa_{yz} \kappa_{zz}^{-1})']^{-1} j_x^2 , \tag{16}$$

$$b = \rho_{xx} j_x^2 \, [(\kappa_{yy} - \kappa_{zy} \kappa_{yz} \kappa_{zz}^{-1})']^{-1} . \tag{17}$$

The temperature distribution along the OY axis acquires the form

$$T = \left[c_1 \exp(\lambda_1 y) + c_2 \exp(\lambda_2 y) - \frac{b}{\lambda_1 \lambda_2} \right]^{0.5} , \tag{18}$$

where λ_1 and λ_2 are the roots of characteristic equation (14). Since, as before, the tensor components π''_{zx} and α'_{xz} are asymptotically equal in absolute value and opposite in sign, the coefficient a of (15) acquires the following form under the given conditions: $a \simeq -(\omega\tau)^3 |\alpha'| \, |\kappa'|^{-1} j_x$, i.e., it is $(\omega\tau)^2$-fold larger than in the previous case, while the coefficient d of (16) retains its previous order of magnitude: $d \simeq -(\omega\tau)^4 |\alpha'|^2 |\kappa'|^{-2} j_x^2$. Since $d/a^2 \simeq (\omega\tau)^{-2}$, then $d_1 \simeq -a$, $\lambda_2 \simeq -d/a$.

As follows from the form of the roots λ_1 and λ_2, the temperature dependence is not a symmetric function of the coordinate y. Since a depends on H and j_x as an odd function, inversion of the directions of the current and the magnetic field must lead to reversal of the temperature distribution, and in this case one face will be heated more strongly than the other. The question of how to determine the integration constants in (18) arises. In the previous case, heat from the sample volume was removed along the axis of the magnetic field and both directions (along and across the OZ axis) were equivalent, $\kappa_{zz} \simeq \kappa$, and the magnetic field affected the heat removal through the off-diagonal components of the tensors of the kinetic coefficients. This provided symmetry, in value, of the corresponding characteristic equation. In the case under consideration the situation is different. Actually, the uniqueness of the solution of second-order equation (14), attainable within the framework of the Cauchy problem, requires that not only the temperature but also its gradient be prescribed at some point. On the other hand, determination of the temperature gradient in the sample volume is a goal of the present problem. Therefore by solving Eq. (14) with Cauchy initial conditions, we may calculate only relative values of the gradients. Within the framework of these constraints we assume that the colder face has the ambient temperature $T(y = l) = T_0$ and the gradient at this point is

$$\left.\frac{\partial T}{\partial y}\right|_{y=l} = \frac{T_0'}{2}.$$

Next, with account for the smallness of the exponent of the root λ_2 the temperature distribution over the sample cross section along the OY axis obeys the following law:

$$T(y) = \left\{ \frac{T_0' T_0}{\lambda_1} \exp\left[\lambda_1 (y - l)\right] + T_0^2 - \frac{T_0' T_0}{\lambda_1} \right\}^{0.5}.$$

It is easily to verify that the ratio of the temperature gradients on the opposite faces is $T'(y = l)/T'(y = -l) \simeq \exp(2\lambda_1 l)$.

The exponential dependence of $\partial T/\partial y$ on j_x and H leads to a complex transformation of the temperature field and the Nernst field under conditions of inversion of the current and the magnetic field, when the gradient $\partial T/\partial y$ on the open face changes by a factor of $\exp(\lambda_1 l)$, while the Nernst electric field even changes sign upon inversion of the magnetic field due to the oddness of the component α_{xy} relative to the field. Moreover, on the adiabatically insulated side the Nernst field is determined by the gradient $\partial T/\partial z$, whose Ettingshausen component is small:

$$\frac{\partial T}{\partial z} = \kappa_{zz}^{-1} \left(\pi_{zx} j_x - \kappa_{zy} \frac{\partial T}{\partial y} \right).$$

We now evaluate quantitatively the degree of mutual influence of galvano- and thermomagnetic phenomena for the particular case where the electric current $I \simeq 10$ A flows through a sample with a cross section of $\sim 10^{-6}$ m^2. For the residual resistance $\rho_0 \simeq 5 \cdot 10^{-13}\ \Omega \cdot$m in the field $H \simeq 3 \cdot 10^7$ A/m the parameters are as follows: $\omega\tau \simeq 1.5 \cdot 10^3$; $\kappa' \simeq \kappa_{zz}' \simeq 5 \cdot 10^4$ W/(m$\cdot$K^2); the parameter characterizing the power of the heat source is $b \simeq 10^3$ K^2/m^2.

Then for $q_y \simeq 0$ the coefficients of characteristic equation (3) are: $a \simeq 10^{-3}$ m^{-1}; $d \simeq -1$ m^{-2}; $\lambda_1 \simeq -\lambda_2 \simeq d^{1/2}$. The gradient $\partial T/\partial z$ of (12) is determined mainly by the heating mechanism, and on the sample surface $\partial T/\partial z \simeq 10^{-1}$ K/m. The Nernst thermoelectric field $\alpha_{xz}(\partial T/\partial z) \simeq 10^{-6}$ V/m, caused by this gradient, is weak compared to the electric field ($\sim 1-10$ V/m) related to the diagonal components of the resistance tensor. At the same time the temperature gradient $\partial T/\partial y$ is determined mainly by the Peltier effect, and the Nernst thermoelectric field is $\alpha_{xy}(\partial T/\partial y) \simeq 10^{-1}-1$ V/m, which amounts to tens of percents of the electric field mentioned above.

In the case of $q_z \simeq 0$ heat is removed in the direction normal to H, and the coefficients of characteristic equation (14) increase ($a \simeq 10^3$ m^{-1}; $d \simeq 1$ m^{-2}), thus causing distortion of the symmetry in the temperature distribution relative to the sample center. Under these conditions $T_0' \le 4 \cdot 10^3$ K/m and the thermoelectric field is $\alpha_{xy}(\partial T/\partial y) \simeq 10^{-2}$ V/m, which amounts to several percent of the field $\rho_{xx} j_x$, while $\partial T/\partial z \simeq 10^{-2}-10^{-3}$ K/m and the corresponding field $\alpha_{xz}(\partial T/\partial z) \simeq 10^{-6}$ V/m is negligible.

To sum up, the anisotropy of heat exchange with the environment that occurs in low-temperature galvanomagnetic investigations may lead to redistribution of the temperature fields over the cross section of metallic samples so that the temperature gradients may be rather high. This circumstance requires taking into account the considered effects for correct evaluation of various thermoelectric fields and determination of the possibility of their elimination from the sought characteristics.

In conclusion, it should be noted that the only definable empirical parameter of the problem in our investigation is the residual resistance of the material, and all other kinetic coefficients are analytically determined from a number of postulates: the approximation of almost free electrons, the elastic scattering of electron, the effective magnetic field, etc. That is why the applicability of the present approach as well as the passage to simpler expressions is determined by an extent and depth of the correspondence of the adopted model to a real physical situation that includes a somewhat larger, than in normal metals, mass of the conduction electrons because of the narrowness of the energy bands, a more complicated form of the collision integral due to the presence of diffusion by magnetic transfer, and possible decompensation of the electron and hole volumes. Moreover, with such lateral dimensions, surface effects exert a certain influence on the kinetics of the charge carriers and, as consequence, on the asymptotic behavior of the tensors of the kinetic coefficients. The obtained temperature distributions over the sample cross section with a tensor nature for the kinetic coefficients describe the mutual influence of galvano- and thermomagnetic characteristics for finite levels of excitation.

NOTATION

H, vector of the magnetic, field intensity; **j**, vector of the current density; **q**, vector of the heat flux; **E**, vector of the electric field; $\hat{\rho}$, tensor of electric resistance; $\hat{\alpha}$, tensor of thermo-emf; $\hat{\kappa}$, tensor of heat conduction; $\hat{\pi}$, tensor describing the Peltier effect; T, sample temperature; E_N, vector of the Nernst thermoelectric field; τ, relaxation time of the carriers; ω, cyclotron frequency.

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